Asymmetric Legislative Bargaining

J. Čopič[1]

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[1]jernej.copic21@gmail.com. Jernej Čopič is at Aix-Marseille Université, a visiting associate at Caltech HSS, and a visiting researcher at CERGE-Ei. I am especially grateful to Duncan Valore-Kemmerer and Jan Zapal for discussions, and also to Yann Bramoulle, Antonio Cabrales, Braz Camargo, Greg Kubitz, Humberto Moreira, Ariel Rubinstein, Jakub Steiner, Vera Zaporozhets, and seminar audiences at FGV-Sao Paulo and Aix-Marseille Université. We are also grateful to N. Hasji, Y. Huang, J. Y. A. Lee and N. Z. A. Teo for assistance.

Abstract

We characterize the equilibrium payoffs in general legislative bargaining environments, where agents differ in discounting and in power to make proposals (recognition probabilities). Our key observation is that under minor restrictions each member of the approving majority must obtain the same share. An approver's share depends on the average interest rate but not on recognition probabilities or approval quota. In contrast with bargaining in markets, an agent's expected rent decreases with patience and is independent of recognition probabilities. Our insight provides a simple closed form, a method for the unrestricted case, and extends to risk aversion and more general settings.

1 Introduction

Almost thirty years have passed since Baron and Ferejohn, (Baron & Ferejohn 1989), published their celebrated legislative bargaining model in which some number of agents bargain over how to divide a budget and an agreement must be confirmed by a majority. Their model has been hugely influential[1] even while restricted to a specific case where all agents are identical. Solving for the equilibrium payoffs in more general legislative bargaining environments has been an open problem ever since.

Baron and Ferejohn model a legislative session where N legislators bargain to allocate a budget between their constituencies. The session proceeds over potentially many periods, $t = 0, 1, ...$, and in each period, one legislator is randomly recognized as the proposer – recognition probabilities p_i are equal for all legislators, $p_i = \frac{1}{N}$. The proposer proposes a split of the budget to the legislature. If a required quota $q < N$ vote in favor of the proposal (e.g., if a simple majority is required, then $q = \frac{N+1}{2}$), then all legislators obtain their proposed shares and the session ends; Otherwise the session moves to the next period where the whole procedure repeats. The legislators are prompted by the opportunity cost of time, which is the same for all legislators, i.e., the discount factor of legislator i is $\delta_i = \delta < 1$ and if the bargaining ends in period t and legislator i obtains a share x, i's payoff is $x\delta^t$. Baron and Ferejohn demonstrated that in a stationary subgame perfect Nash equilibrium, from hereon equilibrium, the session ends in the initial period when the proposer compensates the minimal required number, $q - 1$, of legislators with shares equal to their continuation values $v_i = \frac{\delta}{N}$. These legislators, the approvers, then vote in favor of the proposal along with the proposer; the proposer keeps the

[1]The literature concerning the Baron and Ferejohn model is vast. Apart from other references mentioned elsewhere in the text, we refer to the extensive survey in (Eraslan & McLennan 2013).

residual $1 - \frac{(q-1)\delta}{N}$, and the non-approving members obtain nothing.[2] With its compelling narrative and parsimonious aesthetics, the Baron and Ferejohn model yields a sensible unique equilibrium prediction of how rents are shared in a setting of distributive politics. It is therefore no surprise that it has seen a tremendous success and has become the workhorse for applications in political economy and public finance.

One shortcoming of the Baron and Ferejohn model has been its reliance on the assumption that all legislators are identical, both, in terms of their preferences – discount factors and risk aversion – as well as in the institutional sense – recognition probabilities. Consequently, a strong need emerged early on to extend the model to asymmetric environments. First, such asymmetries arise naturally, not only in real legislative sessions, but also in most other real political settings. For example, when countries bargaining over international agreements, such as trade or pollution quotas, they will face different interest rates in the lending markets, or they may have different relevant horizons, implying different discount factors; and some will have a greater ability to voice their proposals and therefore larger recognition

[2] In rough strokes, a simple derivation is as follows. Assume a symmetric equilibrium. Each i in the role of proposer, which happens with probability $p_i = \frac{1}{N}$, selects approvers with equal probabilities – each legislator is an approver with a probability $\frac{q-1}{N}$. To vote for a proposal, an approver must be at least indifferent – in equilibrium, exactly indifferent – between her proposed share and her continuation payoff v_i should the session proceed to the next period. The proposer compensates $q-1$ approvers, so that the two equilibrium equations are,

$$x^* = \delta\left(\frac{1}{N}x^{**} + \frac{(q-1)\delta}{N}x^*\right) \text{ and } x^{**} + (q-1)x^* = 1.$$

Multiplying the first equation by $\frac{N}{\delta}$ and using the second equation yields $x^* = \frac{\delta}{N}$ and $x^{**} = 1 - (q-1)\frac{\delta}{N}$. In follow-up work, (Eraslan 2002) shows that the equilibrium in (Baron & Ferejohn 1989) is unique up to expected payoffs and (Eraslan & McLennan 2013) prove existence and uniqueness for a more general class of such games.

probabilities than others. Second, studying asymmetric environments is necessary for the comparative statics and policy recommendations for institutional design. A familiar intuition from bargaining environments in market settings is that an agent who is more patient and less risk averse obtains a higher share of the surplus.[3] It seems more than sensible that this intuition should carry over to the political environment. Ever since the inception of the model, these questions have given rise to a sizable literature addressing such asymmetric environments. However, due to technical difficulties inherent to the problem, it has insofar been impossible to provide a closed-form equilibrium characterization, akin to that of the original model, which could provide definite answers and serve as an off-the-shelf tool for applications.[4]

We provide a closed-form equilibrium solution for asymmetric legislative bargaining environments, which is simple and intuitive. The solution delivers comparative statics that are very different from the comparative statics in familiar market environments: a more impatient legislator obtains a higher expected rent, and recognition probabilities (or risk aversion) do not affect the relative expected rents. Our key insight is closely related to perfect competition (or no arbitrage): when the

[3]That is true in the alternating-offers bargaining game, (Rubinstein 1982), and its extensions to multi-lateral settings, e.g., (Krishna & Serrano 1996). To our knowledge, this is also true in essentially any other dynamic bargaining model in a market , under complete or incomplete information. As opposed to political settings considered here, in market settings a deal must be confirmed by all participants.

[4]The sources of these difficulties are the possibly infinite horizon of the bargaining process and the discontinuity in the payoffs to a legislator depending on whether or not she is chosen to be an approver. See the aforementioned (Eraslan & McLennan 2013) for uniqueness and existence results for a more general class of games and (Eraslan 2002) and (Eraslan 2015) for existence and uniqueness results for the present model; the latter and also (Kawamori 2005) provide some comparative statics results regarding the legislators' continuation values; (Kalandrakis 2015) provides a computational algorithm, and (Uyanik 2015) provides a computational algorithm for the more general model of (Eraslan & McLennan 2013).

discount factors and recognition probabilities are not too dissimilar all legislators' shares as approvers must be the same. The intuition is that if, for example, the most impatient legislator required a lower share, then every proposer would want to include her among the approvers, which would increase her discounted continuation payoff and thus her required share. That, in turn, would make her vote more expensive than the votes of the others, a contradiction. Therefore, the implicit competition between potential approvers equalizes all the continuation payoffs and thus all the legislators' shares as approvers. This insight yields the unique equilibrium payoffs of the model: the proposer compensates $q-1$ legislators with $x^* = \frac{\bar{\delta}}{N}$, who then approve the proposal, where $\bar{\delta}$ is given by the discounting associated with the average implied interest rate;[5] the proposer's share is again the residual, $x^{**} = 1-(q-1)\frac{\bar{\delta}}{N}$. A legislator's expected rent prior to the recognition of the proposer is her (un-discounted) continuation payoff $\frac{v_i}{\delta_i} = \frac{x^*}{\delta_i}$. This expression is inversely proportional to i's discount factor and is independent of i's recognition probability – a larger recognition probability is in equilibrium exactly compensated by a smaller probability of being chosen into the winning majority. The legislators' expected rents are also independent of the approval quota q. This reasoning and the comparative statics extend to the environments with risk aversion: in some range, the legislators' shares are independent of their risk aversion.[6]

Our solution applies to a range of parameters that is large enough to allow

[5]That is, if δ_i is the discount factor of legislator i, and ρ_i is the implied interest rate, i.e., $\delta_i = \frac{1}{1+\rho_i}$, then define the average interest rate $\bar{\rho} = \frac{1}{N}\sum_{i \in N} \rho_i$ and $\bar{\delta}$ is the corresponding discount factor, i.e., $\bar{\delta} = \frac{1}{1+\bar{\rho}}$.

[6]At first blush, this might seem at odds with the results in (Kawamori 2005) and (Eraslan 2015), who show that the agents' continuation payoffs are (weakly) increasing with discount factors and recognition probabilities. As our results show, the agents' continuation payoffs are constant over substantial parts of the possible parameter range. The expected rents, which are perhaps more relevant, satisfy the opposite comparative statics.

for meaningful comparative-statics and is suitable for applications where especially the recognition probabilities are not too different. The differences in discount factors would have to be quite large by any realistic standard to fall out of the admissible range. An exception might be the differences in implied discount rates due to different time horizons, e.g., term limits in a legislative setting, or with bargaining over emission quotas, the horizons over which different regions might care for the negative consequences. For environments where differences in parameters are large, our solution does not apply directly but our method does. In some cases, a similar closed-form expression can be derived, for example, when recognition probabilities are very different. That will generally be true in most real legislative settings, and many settings of international negotiations. In Section 4 we derive a closed-form expression for such a setting and discuss some other examples where parameters fall out of the admissible range.

The Baron and Ferejohn model has received widespread attention for good reasons. On the theoretical side, modeling difficulties of political processes are well known and Baron and Ferejohn provided a simple and appealing paradigm with sensible predictions for the case of distributive politics.[7] Their model has consequently been applied to many political settings. In political settings where institutional and preference parameters may be heterogeneous across legislators, the model has either been used directly, or has motivated related bargaining models. Such applications have been made extensively to real legislative environments, as

[7] In general settings, (Arrow 1963) proved impossibility of reasonable aggregation rules, (Gibbard 1973) and (Satterthwaite 1975) proved impossibility of strategy-proof aggregation rules (other than the dictatorial rule), and (McKelvey & Wendell 1976) proved non-transitivity properties of the majority-voting correspondence for multi-dimensional domains; while in single peaked domains the median voter theorem holds in a one-dimensional setting, (Plott 1967) proved a generic impossibility of similar results for multi-dimensional settings. Allocating a budget between many legislators is a specific case of such a multi-dimensional single-peaked preference domain.

in e.g., (Knight 2008), (Berry, Burden & Howell 2010), (Albouy 2013), (Bonatti & Thomson 2015), and many others; or to bargaining over international agreements, as in (Tarar 2005), (McLean & Stone 2012), (Bagwell, Staiger & Yurukoglu 2014), and (Bowen 2015). The model has been used as a testbed in laboratory environments, e.g., (Fréchette, Kagel & Lehrer 2003), (Fréchette, Kagel & Morelli 2005*a*), (Fréchette, Kagel & Morelli 2005*b*), (Agranov & Tergiman 2014), (Miller & Vanberg 2015), and (Nunnari 2016). The legislative bargaining model has also been used as a building block for more complex political models, as in and (Snyder, Ting & Ansolabehere 2005), (Imai & Salonen 2012), and (Battaglini, Nunnari & Palfrey 2012).

The present closed form solution offers a simple and natural model that can serve for such empirical, experimental and theoretical applications. It also offers a testable comparative-static prediction, which has not been known before, and which clearly distinguishes the political environment presented here from a market setting. In more general political economy environments, where agents may also have different voting weights, related models (notably (Eraslan & McLennan 2013)) have been used for applications such as corporate bankruptcy, (Earslan 2008), French water negotiations (Thomas & Zaporozhets 2016), and Belgian railway negotiations, (Proost & Zaporozhets 2013), and to estimate the effect of voting power, . Beyond pure theoretical curiosity, there are therefore good reasons to study asymmetric environments. The key insight here, that the continuation values of many agents will in equilibrium be equal, will largely carry over to these environments and facilitate solving these models explicitly.

In the sense of economic intuition we feel that our main contribution here are the comparative static results. In particular, in political settings the more impatient agents may obtain a larger share of surplus. This can be restated as a distinctive property of bargaining power in different settings: in a political setting bargaining power is inversely proportional to an agent's discounting, whereas in a market setting bargaining power is proportional to agent's discounting. More generally, our results suggest that power in political settings depends on preference and institutional parameters very differently than the bargaining power in markets, which offers some testable predictions of the model. One could also apply our results to determine some key parameters of legislative decision making – see (Kauppi & Widgren 2004) and (Zaporozhets, García-Valiñas & Kurz 2016) for related examples of such empirical analysis, however, these works are not based on the non-cooperative legislative bargaining model considered here. Beyond the comparative static result, the intuitive and portable closed-form expression for the realistic bargaining framework should therefore facilitate further applications in theory and practice.

In the next section we present the formal intuition when agents differ only in their discount factors. In Section 3 we present the general results, and in Section 4 we discuss examples where heterogeneity among the agents is large.

2 Formal intuition

To begin with, assume that all agents are risk neutral, i.e., the utility of agent i if she obtains a share x in period t is $\delta_i^t x$. As is standard in such games, what we mean by equilibrium is a stationary subgame perfect Nash equilibrium, where weakly dominated strategies are eliminated at the voting stage. It has been shown that in equilibrium such a game will end in the first period and the payoffs will be uniquely determined, see (Eraslan & McLennan 2013), who call this a reduced equilibrium and we refer to their work for the general definition of equilibrium.[8]

[8]The model here is very similar to a special case of (Eraslan & McLennan 2013), who allow for different voting weights. One difference is that in (Eraslan & McLennan 2013) agents submit

Our main result is formally quite simple, and to make the intuition as transparent as possible, we first give the argument for the case when the legislators differ only in their discount factors. In the original Baron and Ferejohn model, the driving force behind the equilibrium is the proposer's desire to collect as high rents as possible under the constraint of getting her proposal approved. A proposer will compensate the minimal (or cheapest) majority of the legislators to approve her proposal. Since all the legislators are the same, they will on average be chosen as approvers with equal likelihoods and as approvers all will obtain the same shares equal to their continuation payoffs. Now suppose, for example, that legislator 1 is less patient than the other legislators, $\delta_1 < \delta_2 \leq ... \leq \delta_N$. As argued in the introduction, if the more impatient legislator 1 had a lower continuation payoff, then she would be picked as an approver with certainty, and provided that the discount factors were not too different, her continuation payoff would indeed be higher, a contradiction.

The way out of this riddle is to realize that in equilibrium all legislators' shares as approvers must be the same. That is achieved by the proposers' randomization over their choice of approvers from amongst the other legislators.[9] When the discount factors are not too different, a more impatient legislator is an approver more often than the more patient legislators. Denote by $\bar{\mu}_i$ the (average) probability

their votes sequentially whereas here the votes are submitted simultaneously. Any equilibrium of the sequential voting model will be an equilibrium under simultaneous voting so that with that modification, the equilibrium definition and existence and uniqueness results carry over to the present model. If one does not restrict attention to stationary equilibria, then any shares can be supported in a subgame perfect equilibrium. Eliminating weakly dominated strategies at the voting stage is necessary to eliminate outcomes where no agent votes for the proposal simply because no other agent votes for the proposal.

[9]This is standard in games with discontinuous payoffs and it is necessary to assure that the best-reply correspondence is convex, see e.g., (Simon 1987), (Simon & Zame 1990), and (Reny 1999)

with which i is chosen as an approver by the other proposers, e.g., legislator 1 above is chosen as an approver by the other proposers with a probability $\bar{\mu}_1 \in (0, 1 - \frac{1}{N})$, and $\bar{\mu}_i < \bar{\mu}_1$. Each proposer needs $q - 1$ approvers, which yields the voting-quota constraint,

$$\sum_{i=1}^{N} \bar{\mu}_i = q - 1. \tag{1}$$

All approvers obtain the same share x^*, which equals their discounted continuation payoffs, and consequently every proposer obtains the same residual share $x^{**} = 1 - (q-1)x^*$, which is the resource constraint,

$$(q-1)x^* + x^{**} = 1. \tag{2}$$

Legislator i's continuation payoff is given by her expected rent she gets as the proposer plus her expected rent she gets as an approver, i.e., $\frac{1}{N}x^{**} + \bar{\mu}_i x^*$. As in the Rubinstein's alternating-offers game, in order to approve the proposal, a legislator must be indifferent between her share x^* and her discounted continuation payoff, which yields the equilibrium approval conditions,

$$x^* = \delta_i(\frac{1}{N}x^{**} + \bar{\mu}_i x^*), i \in N. \tag{3}$$

Dividing these by δ_i, and summing over all legislators,

$$x^* \left(\sum_i \frac{1}{\delta_i} \right) = x^{**} + x^* \left(\sum_i \bar{\mu}_i \right) = 1,$$

where the last equality follows from the voting-quota constraint (1) and the resource constraint (2). The solution for x^* follows by noting that $\sum_i \frac{1}{\delta_i} = \frac{N}{\bar{\delta}}$, where $\delta_i = \frac{1}{1+\rho_i}$ and $\bar{\delta} = \frac{1}{1+\bar{\rho}}$, with $\bar{\rho} = \frac{1}{N}\sum_i \rho_i$.

What remains is to verify the conditions under which the legislators' discounted continuation payoffs are equal. For the argument to be valid for the most impatient legislator, δ_1 must satisfy that if 1 is chosen into the winning majority whenever she is not a proposer, her discounted expected continuation payoff should be no less than x^*. That is, $x^* \leq \delta_1(\frac{1}{N}x^{**} + \frac{N-1}{N}x^*)$. Similarly, if the most patient legislator N is never chosen into the winning majority, her expected continuation payoff should not be greater than x^*, i.e., $x^* \geq \frac{\delta_N x^{**}}{N}$. These two conditions describe the admissible range of δ_i's,

$$\frac{\delta_N x^{**}}{N} \leq x^* \leq \delta_1(\frac{1}{N}x^{**} + \frac{N-1}{N}x^*).$$

When the legislators differ in their recognition probabilities, the equilibrium expression and derivation remain essentially the same. We present that next, along with a few details that were omitted here.

3 General result

In general, consider the following conditions (4) and (5),

$$\frac{\bar{\delta}}{N - q\bar{\delta}} \geq \max_i \delta_i p_i \tag{4}$$

$$\frac{\bar{\delta}}{N} \leq \frac{1}{N} \min_i \delta_i \left(\bar{\delta} + p_i(N - q\bar{\delta})\right) \tag{5}$$

The two conditions constrain the maximal differences between the agents' discount factors and recognition probabilities. These conditions can be rearranged into, $\max_i \delta_i p_i x^{**} \leq x^* \leq \min_i \delta_i(p_i x^{**} + (1 - p_i)x^*)$. Thus, condition (4) states that the maximal discounted continuation payoff of an agent is low enough when she is never chosen as an approver, that is, in equilibrium it is not higher than that of the rest. Condition (4) is that the minimal discounted continuation payoff of an agent is not too low even if she is always chosen as an approver, that is, in equilibrium, it is at least as high as that of the rest.

Recall that v_i denotes the continuation payoff to agent i if the current period proposal were not approved and the bargaining game were to proceed to the next period.[10] We now assume that these continuation payoffs are equal for all legislators, $v_i = v^*, \forall i$, construct an equilibrium under that assumption, and then verify that the assumption holds under the above conditions (4) and (5).

In equilibrium, a proposer j selects $q-1$ other agents, and compensates them with shares that are large enough for these to vote in favor of the proposal. By familiar arguments, for each of these agents, these shares must in equilibrium be equal to v^*. Consequently, whenever selected as an approver, each agent obtains the same share, denoted by x^*. This implies that every proposer obtains the same share, denoted by x^{**}, where x^{**} is given by the resource constraint (2), $x^{**} = 1-(q-1)x^*$.

Theorem 1. *If and only if the recognition probabilities and the discount factors satisfy conditions (4) and (5), the unique equilibrium payoffs are given by,* $x^* = \frac{\bar{\delta}}{N}$ *and* $x^{**} = 1 - \frac{(q-1)\bar{\delta}}{N}$*, and the game ends in the first period. The expected rent to agent* i *is* $\frac{x^*}{\delta_i} = \frac{1}{N}\frac{\bar{\delta}}{\delta_i}$.

Proof. Since each proposer is indifferent between selecting any set of $(q-1)$ approvers, she can randomize over such sets. Denote by $\mu_{i,j}$ the probability with which j as proposer selects i as an approver. Since each proposer j needs $(q-1)$ approvers, we have for each j the voting quota constraint,[11]

$$\sum_{i \neq j} \mu_{i,j} = q-1, \forall j. \tag{6}$$

In equilibrium, an agent's continuation value is given by her discounted expected rent, which is $\delta_i(p_i x^{**} + \sum_{j \neq i} p_j \mu_{i,j} x^*)$. Since $v_i = v^* = x^*, \forall i$, we

[10] Formally, a continuation payoff of agent i is indexed by the period, i.e., $v_{i,t}$, however, in equilibrium by stationarity, $v_{i,t} = v_i, \forall t$.

[11] The probabilities $\mu_{i,j}$ are not independent as they are derived from probabilities over different sets of approvers of size $q-1$ – the proposer randomizes over approvers but always allocates shares x^* to precisely $q-1$ approvers.

obtain the approval condition,

$$x^* = \delta_i(p_i x^{**} + \sum_{k \neq i} p_k \mu_{i,k} x_{i,k}), i \neq j. \tag{7}$$

When continuation values of all agents are the same, an equilibrium is therefore given by (2),(3), and (6).

Let $\bar{\mu}_i = \sum_{j \neq i} p_j \mu_{i,j}$, i.e., as in the introduction, $\bar{\mu}_i$ denotes the average likelihood with which i is chosen as an approver. Therefore,

$$\sum_i \bar{\mu}_i = \sum_i \sum_{j \neq i} p_j \mu_{i,j} = \sum_j p_j \sum_{i \neq j} \mu_{i,j} = q - 1.$$

Next, assume that the discounted expected continuation payoffs are equal to x^* for all agents, and the proposer's share x^{**} is equal for all agents, so that (7) becomes,

$$x^* = \delta_i(p_i x^{**} + \bar{\mu}_i x^*), \forall i.$$

As in Section 2, this gives the equilibrium shares,

$$x^* = \frac{\bar{\delta}}{N}, \ \ x^{**} = 1 - (q - 1)\frac{\bar{\delta}}{N}.$$

For the above argument to be true, the highest discounted expected continuation payoff of an agent should not be strictly more than her compensation x^*, when she is never included in the winning majority. That is,

$$x^* \geq \max_i \delta_i p_i x^{**},$$

which yields condition (4). Similarly, the lowest discounted expected continuation payoff of an agent should not be strictly less than her compensation x^*, when she is included in the winning majority by every proposer. That is,

$$x^* \leq \min_i \delta_i(p_i x^{**} + (1 - p_i)x^*),$$

which yields condition (5). The proof now follows by the uniqueness of equilibrium payoffs, see (Eraslan & McLennan 2013). □

The logic of the argument extends to the case where the agents risk averse, i.e., when the utility function of agent i is given by $\delta_i^t u_i(x)$, where u_i is strictly increasing and concave (to give appropriate restrictions analogous to (4) and (5) one might further assume that u_i is differentiable). In that case it is in general not possible to obtain a closed-form expression for the equilibrium shares. However, it is still possible to show that under certain conditions, all agents' shares as approvers must coincide, i.e., equal to some x^*. As before, the expected rent of agent i is given by $\frac{x^*}{\delta_i}$. Therefore, under those conditions, everything else equal, differences in risk aversion do not imply any differences in the agents' expected rents.

More precisely, under risk aversion, the approvers' equilibrium share x^* is given as the solution to the system,

$$\frac{x^*}{\delta_i} = p_i u_i(1 - (q-1)x^*) + \bar{\mu}_i u_i(x^*), i = 1, ..., N.$$

$$\sum_i \bar{\mu}_i = q - 1,$$

if and only if, such x^* satisfies the two conditions, $x^* \leq \min_i u_i(1 - (q-1)x^*) + \bar{\mu}_i u_i(x^*)$, and $x^* \geq \max_i u_i(1 - (q-1)x^*)$.

4 Further examples

Finally, one may ask whether some of the intuitions of the present approach continue to hold when (4) and (5) are not satisfied. That is indeed the case. Effectively, Theorem 1 presents the case of interior solutions to the problem. When one or the other condition is not satisfied, the problem has a corner solution whereby some of the legislators are never chosen as approvers, and some are chosen as approvers with certainty by all proposers. However, the approvers' shares of all the legislators for whom both conditions are met are still equal and for those legislators the comparative statics results still hold as well. We first present an application where (4) is not met.

It is quite common in legislative environments that some legislators will have greater access to the floor than others. For example, in the US congress, such legislators are the leader of the majority, or the speaker, or the chairmen of specific committees.[12] Consequently these legislators will have much larger recognition probabilities than the rest of the legislature. The continuation payoffs of such legislators will then be relatively large even if they are never included in the winning majority so that condition (4) will not hold for them. Such is plausible for many other applications, for example, international agreements between countries.

Suppose then that there are some relatively small subset I of the agents, $|I| < N - q$, for whom (4) is violated, and let J be the set of agents who satisfy both conditions, $J = N \setminus I$. The agents in I will then never be included in the winning majority as their votes are too expensive and each proposer can form a winning majority out of agents in J. The agents in J are included proportionally more often, more precisely,

$$\sum_{i \in J} \bar{\mu}_i = q - 1, \text{ and, } \sum_{i \in I} \bar{\mu}_i = 0.$$

Now denote the approvers' share by x_J^*. Since there are sufficiently many "cheap" approvers to satisfy the voting quota q, the proposer's share x^{**} is still the same for all proposers, $x^{**} = 1 - (1 - q)\bar{x}^*$, $\forall i \in N$. Moreover, for all agents in J their approvers' shares are still equal to their expected discounted continuation payoffs,

$$x_J^* = \delta_i (p_i x^{**} + \bar{\mu}_i x_J^*), \forall i \in J.$$

[12] This is related to the motivation in (Copic & Katz 2012) and (Copic & Katz 2014), who consider a static model of agenda setting that is built on the assumption that one agent, e.g., the speaker, has exclusive rights to the floor.

Denote $p_J = \left(\sum_{i \in J} p_i\right)$ and let $\bar{\delta}^*_J$ be the discounting corresponding to the average interest rate of the agents in J. Solving for the equilibrium payoffs (as in the Theorem 1), we obtain,

$$x^*_J = \frac{\bar{\delta}^*_J p_J}{|J| - \bar{\delta}^*_J (1 - p_J)(q - 1)}$$

In this case only the discount factors of the less powerful, or "relatively cheap" agents matter for the equilibrium shares. Note that the conditions that must be satisfied by the legislators in J are now slightly different from (4) and (5), i.e., they can be written as,

$$\delta_i p_i x^{**} \leq x^*_J \leq \delta_i (p_i x^{**} + (1 - p_i) x^*_J), \forall i \in J.$$

For legislators in $i \in I$, the condition is that $\delta_i p_i x^{**} \geq x^*_J$. When $I = \emptyset$ then $p_J = 1$, and x^*_J equals x^* from Theorem 1.

Thus, when a small number of agents are much more powerful than the rest, our key idea still applies to the less powerful agents. There is an adjustment of the equilibrium approvers' shares which is proportional to the number of the more powerful agents. The adjustment is equivalent to a legislative environment where in each period with some probability no proposer is selected, so that the example above delivers a closed-form expression for that case – apart from different voting weights, that is another generalization of the model in (Eraslan & McLennan 2013). An extreme example of such an environment is when there is one proposer with almost exclusive rights to the floor, e.g., $p_1 >> p_i, i > 1$. The other agents might then still obtain non-negligible rents, depending on $p_J = 1 - p_1$ and their discount factors.

The comparative statics evidently do not change for the agents in J. The expected rents of agents in I might now potentially be larger, which is due entirely to their larger recognition probabilities. Everything else equal, their expected rents are still inversely proportional to their discount factors.

For an illustration when (5) fails, consider a setting where all recognition probabilities are similar but legislator 1 is quite impatient relative to the rest. Then, even if 1 were always included in the approving majority when she is not a proposer, she would still receive a lower discounted expected payoff than the rest. Every other proposer will need $q-2$ other approvers (apart from herself and legislator 1), while legislator 1 as a proposer will need $q-1$ relatively expensive approvers so that 1 should be considered differently from the rest and her share as a proposer will also be lower than the others'. No new difficulties arise if more than one legislator is very impatient, but one must keep careful book-keeping of what legislators different proposers select for approvers and with what shares. Our main intuition still applies to the more patient legislators whose continuation payoffs must in equilibrium coincide.[13]

To put some flesh on the bones, consider an example with $N = 3$ where all legislators have the same recognition probabilities, $p_i = \frac{1}{3}$, and assume that two patient legislators have the same discount factor, i.e., $\delta_1 < \delta_2 = \delta_3$. Then (5) is violated if $\frac{1}{\delta_1} - \frac{1}{2} > \frac{1}{\delta_2}$, which is for most applications a large difference in discount factors. The equilibrium conditions are then,

$$
\begin{aligned}
x_1^* &= \delta_1 \left(p_1 x_1^{**} + (1-p_1) x_1^*\right), \\
x_1^{**} &= 1 - x_2^*, \\
x_2^* &= \delta_i \left(p_i x_2^{**} + \mu_i x_2^*\right), i = 2, 3, \\
x_2^{**} &= 1 - x_1^*.
\end{aligned}
$$

In equilibrium proposers 2 and 3 choose agent 1 for sure, while 1 randomizes equally between 2 and 3, so that $\bar{\mu}_1 = \frac{2}{3}$, $\bar{\mu}_2 = \bar{\mu}_3 = \frac{1}{6}$ and we obtain x_1^* and

[13] In general, when the legislators are risk neutral, the equilibrium shares are characterized as a solution to a system of linear equations, the size of which depends on the number of legislators with low continuation values.

x_2^* after some manipulation. As should be the case, (5) is violated precisely when $x_1^* < x_2^*$.[14]

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[14]That is, $x_1^* = \frac{2\delta_1 - \delta_1\delta_2}{6 - 4\delta_1 - \delta_2}$ and $x_2^* = \frac{2\delta_2 - 2\delta_1\delta_2}{6 - 4\delta_1 - \delta_2}$. For example, when $\delta_1 = \frac{1}{2}$ and $\delta_2 = \delta_3 = \frac{3}{4}$, then $x_1^* = \frac{5}{26}$ and $x_2^* = \frac{6}{26}$.

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